Ultimo risk measures for surrender risk and risk adjustment in IFRS17

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Risk adjustment in IFRS17

Paragraph 37: ”An entity shall adjust the estimate of the present value of the future cash flows to reflect the compensation that the entity requires for bearing the uncertainty about the amount and timing of the cash flows that arises from non-financial risk.”

Paragraph 119: ”An entity shall disclose the confidence level used to determine the risk adjustment for non-financial risk. If the entity uses a technique other than the confidence level technique for determining the risk adjustment for non-financial risk, it shall disclose the technique used and the confidence level corresponding to the results of that technique.”

Methods for risk adjustment include

- the cost of capital method - similar to the risk margin in Solvency II
- the percentile method - treated in this talk

Goal of today's talk

One of the main non-financial risks is Surrender risks, also called Lapse risk.

I will propose a risk adjustment for Surrender risk, assuming "normally distributed" increments of surrender rates. The risk adjustment will be a quantile (or a partial expectation) for the present value of future cash flows, a so-called ultimo risk measure.

Limitations: 1) Mass lapse events are not considered, 2) Homogeneity, 3) Negative surrender rates are possible, 4) One risk only!

Notation

Assume a portfolio of insurance contracts whose future net cash flows are a_t for each year $t = 1, 2, ..., T$, given a surrender rate of zero. Each cash flow is the net of the premium income minus insurance outpayments and expenses. They are based on actuarial assumptions and should be seen as already discounted. The PVFCF from the portfolio is

$$\sum_{t=1}^{T} a_t.$$

We will model remain rate instead of the surrender rate; let its best estimate be r at time 0. I.e. the expectation of the remain rate during the first year is r.

Process r_t, where for each year t, a proportion of contracts equal to r_t remain in the portfolio.

Notation, cont.

Simplifying assumtion of homogeneity: all contracts have the same probability r_t to remain at time t. We get a modified cash flow at time t:

$$b_t = a_t \prod_{s=1}^{t} r_s.$$

The total PVFCF for the portfolio becomes

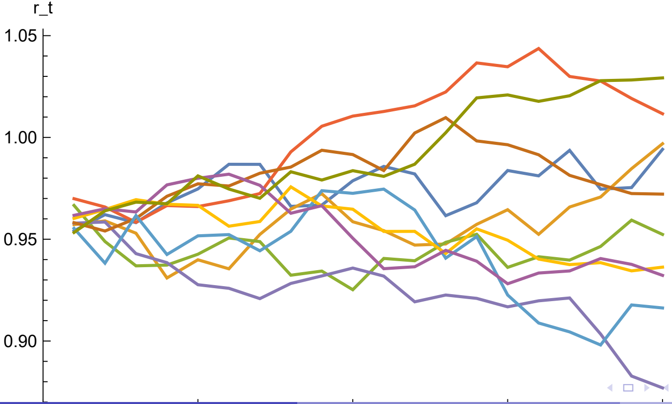
$$S = \sum_{t=1}^{T} b_t.$$

A stochastic model for surrenders

Assume that

$$r_t := \begin{cases} re^{-\sigma^2/2+\sigma X_1} & \text{if } t = 1, \\ r_{t-1}e^{-\sigma^2/2+\sigma X_t} & \text{if } t \geq 2, \end{cases}$$

where X_t are standard normal and independent variables.
Note that r_t is a Markov process, and $E[r_t] = r_{t-1}$, i.e. r_t is a martingale. Examples of remain rates (r=0.96):



Example of cash flow projections

Set $a_t = 1$ for all t.

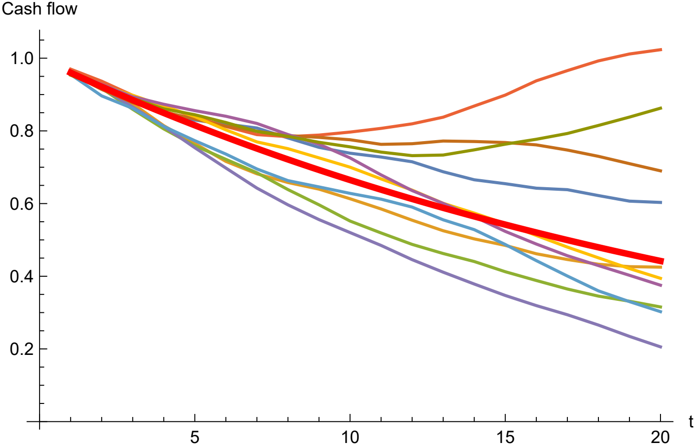


Fig. 1

Note that the thin red line is an artefact of the model. With $\sigma = 0$ we get the deterministic cash flows 0.96^t (thick red).

A stochastic model for surrender, cont.

We have, using the stochastic iteration,

$$r_s = re^{-s\sigma^2/2+\sigma(X_1+\ldots+X_s)}.$$

Hence

$$\begin{aligned} b_t &= a_t \prod_{s=1}^{t} r_s = a_t \prod_{s=1}^{t} re^{-s\sigma^2/2+\sigma(X_1+\ldots+X_s)} \\ &= a_t r^t e^{-t(1+t)\sigma^2/4+\sigma(tX_1+(t-1)X_2+\ldots+X_t)} \\ &= a_t r^t e^{-t(1+t)\sigma^2/4+\sigma Z_t}, \end{aligned} \tag{1}$$

where $Z_t = \sum_{i=1}^{t}(t-i+1)X_i$. Note that b_t is lognormal.

Term structure of surrender rates (Lina Balčiūnienė)

Assume that the stochastic remain rate in year s is

$$r_s = r(s)e^{-s\sigma^2/2+\sigma(X_1+\ldots+X_s)},$$

where $r(s)$ are given for each future s. Then

$$\begin{aligned} b_t &= a_t \prod_{s=1}^{t} r_s = a_t \prod_{s=1}^{t} r(s)e^{-s\sigma^2/2+\sigma(X_1+\ldots+X_s)} \\ &= a_t \left(\prod_{s=1}^{t} r(s)\right) e^{-\sigma^2 t(t+1)/4+\sigma(tX_1+(t-1)X_2+\ldots+X_t)} \\ &= a_t \left(\prod_{s=1}^{t} r(s)\right) e^{-\sigma^2 t(t+1)/4+\sigma Z_t}. \end{aligned} \tag{2}$$

Risk measures for individual future cash flows

Each Z_t is a normally distributed variable with mean 0, and their covariance matrix C is:

$$C_{s,t} := \operatorname{cov}[Z_s, Z_t] = \sum_{i=1}^{t} (t-i+1)(s-i+1) = \frac{t(t+1)(3s-t+1)}{6},$$

where $t \leq s$. In particular, the variance of Z_t is

$$V[Z_t] = C_{t,t} = \frac{t(t+1)(2t+1)}{6}.$$

Hence the p-quantile of b_t equals

$$Q_p[b_t] = a_t r^t \exp\left(-t(1+t)\sigma^2/4 + \sigma z_p \sqrt{\frac{t(t+1)(2t+1)}{6}}\right),$$

where $z_p = \Phi^{-1}(p)$ is the p-quantile of the standard normal distribution. This is a stressed PVFCF for time t and a low quantiles (e.g. $p = 0.2$), and a typical risk adjustment is the difference between the deterministic cash flow ($\sigma = 0$) and the quantile.

Risk measures for individual future cash flows, cont.

Let us also define the p-Partial expectation of X as the average of the quantiles

$$\mathrm{PE}_p[X] = \frac{1}{p}\int_0^p Q_q[X]dq.$$

In our case,

$$\mathrm{PE}_p[b_t] = \frac{a_t r^t}{p}\exp\left(\frac{\sigma^2 t(t+1)(t-1)}{6}\right)\Phi\left(z_p - \frac{\sigma t(t+1)(2t+1)}{6}\right).$$

Risk adjustment for the whole portfolio. Aggregation

We want to make the risk adjustment for the portfolio as a whole, so how do we aggregate?

"Sum of quantiles (of cash flows)":

$$\sum_{t=1}^{T} Q_p[b_t].$$

This is easy, conservative, and basically assumes all the cash flows perfectly dependent. (Comonotonicity)

"Quantile of sums (of cashflows)":

$$Q_p\left[\sum_{t=1}^{T} b_t\right].$$

This is more correct, but requires simulation. Or some approximation, to what we now turns.

Convex ordering of stochastic variables

A function f is convex if $f(tx_1 + (1-t)x_2) \leq tf(x_1) + (1-t)f(x_2)$ for all t between 0 and 1, and all x_1, x_2. Example x or x^2.

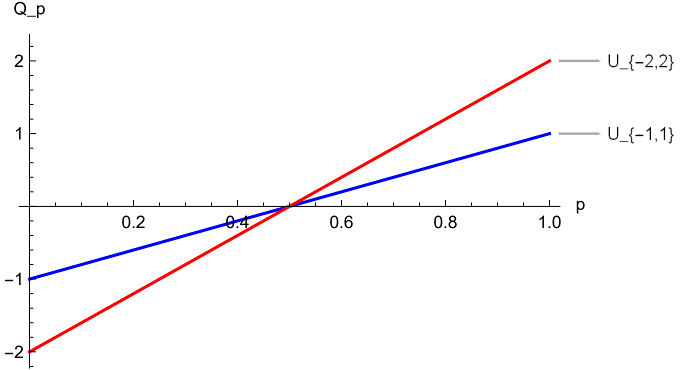
A stochastic variable X is said to precede Y in the convex order sense, notation $X \leq_{cx} Y$, if

$$E[v(X)] \leq E[v(Y)]$$

for all convex functions v. This basically means that the tails of Y go further out from the centre than the tails of X.

Convex ordering of stochastic variables, cont.

Example: $U_{\{-1,1\}} \leq_{cx} U_{\{-2,2\}}$.



Theorem: Convex ordering preserves Partial expectation: if $X \leq_{cx} Y$, then $PE_p[X] \geq PE_p[Y]$ for all p. (But not always quantiles.)

Convex bounds for sums of stochastic variables

Let U be uniform on (0,1). For any stochastic variable X, $F_X^{-1}(U)$ has the same distribution as X. Proof:
$P(F_X^{-1}(U) < x) = P(U < F_X(x)) = F_X(x)$.

Let $(X_1, \ldots, X_n)$ be any random vector, and U uniform on (0,1). Then

$$\sum_{i=1}^{n} X_i \leq_{cx} \sum_{i=1}^{n} F_{X_i}^{-1}(U).$$

The right side of the inequality is the comonotonic counterpart of the left side; each term has the same marginal distribution as the corresponding term at the left. But they are perfectly dependent: we use a single U. In our application, the right bound is the "sum of quantile (of cash flows)".

Convex bounds for sums of stochastic variables, cont.

Let $(X_1, \ldots, X_n)$ be any random vector,and Λ any random variable. Then

$$\sum_{i=1}^{n} E[X_i|\Lambda] \leq_{cx} \sum_{i=1}^{n} X_i.$$

This bound is very useful, in particular when Λ resembles $S := \sum X_i$ in some way. Typically one uses $\Lambda = \sum \beta_i X_i$, for a choice of coefficients β_i.

Theorem: Convex bounds for sums of lognormal variables

$$S = \sum_{t=1}^{T} \alpha_t \exp(Y_1 + \ldots + Y_t),$$

where $(Y_1, \ldots, Y_T)$ has a multinormal distribution, and $\alpha_t \geq 0$. Set $Y(t) = Y_1 + \ldots + Y_t$, and define

$$S_l := \sum_{t=1}^{T} \alpha_t \exp\{E[Y(t)] + \rho_t \sigma_{Y(t)} \Phi^{-1}(U) + \frac{1}{2}(1 - \sigma_t^2)\sigma_{Y(t)}^2\}.$$

$$S_u := \sum_{t=1}^{T} \alpha_t \exp\{E[Y(t)] + \sigma_{Y(t)} \Phi^{-1}(U)\}.$$

Then

$$S_l \leq_{cx} S \leq_{cx} S_u.$$

Note that S_u corresponds to the sum of quantiles.

Example ($T = 60$, $a_t = 1$ for all t, $r = 0.96$, $\sigma = 0.01$

Graph of approximations of quantiles of the total PVFCF:

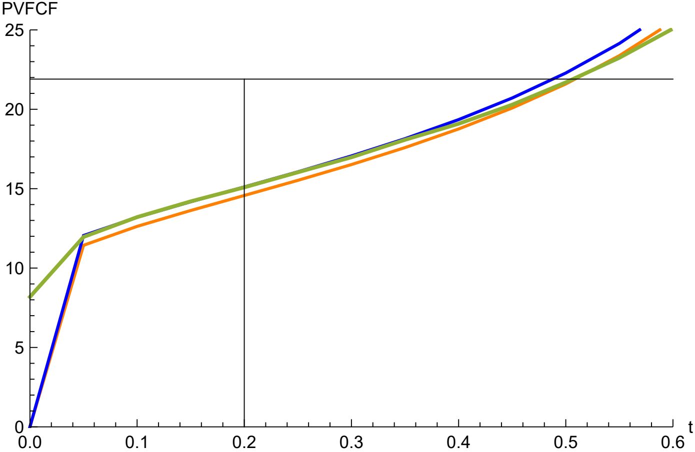


Fig. 4

The orange line is S_u, the blue line is S_l, and the thicker green line is S.

Example, cont.

Graph of approximations of partial expectation of the total PVFCF:

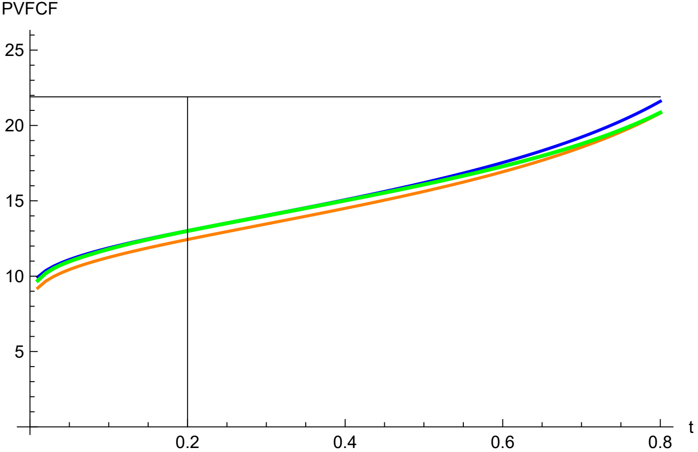


Fig. 5

The orange line is S_u, the blue line is S_l, and the thicker green line is S.