The Danish partial internal model for longevity risk

Kristian Buchardt

Head of Innovation & Models
Finance & Actuarial Department
PFA

Members meeting at the Swedish Society of Actuaries
Stockholm, April 11, 2019

Contents

Contents

Danish life- and pension insurance

Danish products: Can be categorized in two parts:

- Guaranteed pensions with profit sharing (e.g. life annuities)
- Unit-linked products

Situation: Market-based valuation of assets and liabilities

- The balance is identical in DK GAAP and Solvency II
  - The value of the current guaranteed benefits
  - Future discretionary bonus - (currently) calculated residually
- **Financial risks:** Partly controlled via investments/derivatives
- **Mortality and longevity risk:**
  Essential **non-hedged**, **non-hedgeable** risk for pension funds

Solvency II:

- Capital requirements derived from inherent risks and investments
- **Longevity risk:** In standard model, 20 percent reduction in mortality rates (**possibility for internal models**)

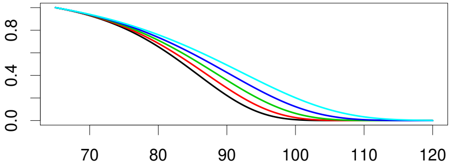
The importance of the trend

Key issues:

- Current mortality
- Expected future mortality development (**the trend**)

Illustration of importance of future trend for valuation

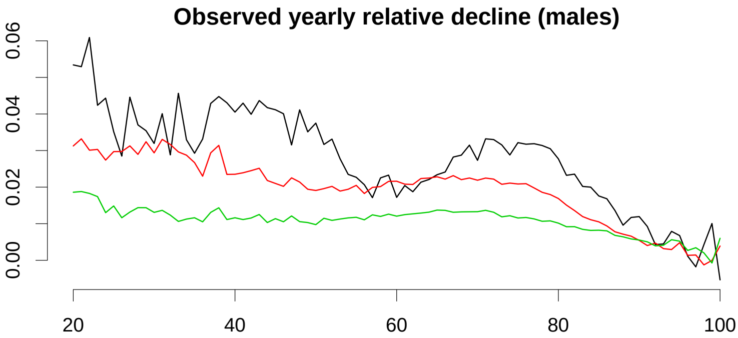
Survival probability (male, initial age 65)



Trend	V	E	${}_{25}p_{65}$
0%	14.80	18.2	22.2%
1%	15.54	19.4	28.8%
2%	16.38	20.7	35.7%
3%	17.33	22.3	42.6%
4%	18.43	24.3	49.2%

(**Trend 0%, 1%, 2%, 3%, 4%**)

Improvement rates for different periods



Period: (**2000-2018**, **1980-2018**, **1960-2018**)

Difficult to predict future changes:

- Yearly improvement rates depend on the period considered
- Largest improvement rates in the recent period 2000-2018

(Statistics Denmark and Human mortality database, www.mortality.org)

Highlights of the history of the partial internal model

- 2010 The Danish FSA establishes the mortality benchmark in december
- 2012 Working group at the Danish Society of Actuaries about a partial internal model
- 2013 Working group with individual companies via Insurance & Pension Denmark
- 2015 Changes to the model as part of dialogue with FSA during the approval process
- 2015 **Approval of the partial internal models under Solvency II**
- 2017 The Danish FSA changes the trend in the mortality benchmark (30 years to 20 years)
- 2018 Changes of certain (technical) aspects of the partial internal model

Contents

The Danish FSA benchmark mortality model

The Danish FSA introduced in December 2010 a **mortality and longevity benchmark**:

1. Benchmark for **current mortality** $\mu^{FSA}(x, t_0)$ estimated from insured population with **5 years of data**
2. Benchmark for **future trend** $R(x)$ estimated from total Danish population with **~~30~~ 20 years of data**
3. Pension funds perform yearly statistical tests whether they deviate from benchmark:
   Estimate $\beta = (\beta_1, \beta_2, \beta_3)$. Same level after age 100.

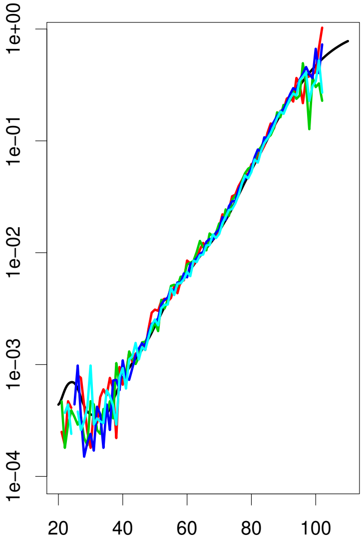
Mortality model:

$$\mu_\beta(t, x) = e^{\beta_1 r_1(x) + \beta_2 r_2(x) + \beta_3 r_3(x)} \cdot \mu^{FSA}(x, t_0) \cdot (1 - R(x))^{t - t_0}$$

FSA benchmark: Current level of mortality

Current mortality:
$\mu^{FSA}(x, t_0)$

- Estimated from 5 years of data for the insured population
- Log-linear regression over time and kernel smoothing over age
- Special model for high age mortality - logistic model
- Non-exponential increase of mortality after age 100

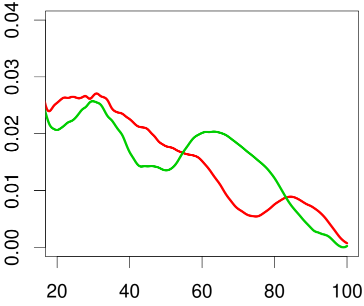


FSA benchmark: Future mortality improvements (trend)

Mortality improvements (trend): $R(x)$

- Based on the whole Danish population
- Future trend: average of the last ~~30~~ 20 years trend
- Kernel smoothing applied
- Age- and gender-dependent yearly decline

Current mortality $\mu^{FSA}(x, t_0)$ and trend $R(x)$ lead to best estimate



Trend: **females**, **males**

Expected life times in 2019

Age	Males	Females
40	88,9	90,6
50	87,8	89,7
60	87,2	89,1
70	87,5	89,1
80	89,3	90,6
90	94,1	94,9

FSA benchmark: company-specific mortality

Two basic questions:

- Is portfolio-specific mortality equal to benchmark mortality?
- What is "best estimate" of portfolio specific mortality?

Notation: Portfolio mortality data for age x and year t

- $D(x, t)$ number of deaths year t and age $[x, x+1)$
- $E(x, t)$ total exposure year t and age $[x, x+1)$

Model: All D's independent with

$$D(x, t) \sim \text{Poisson}(\mu_\beta(x, t)E(x, t))$$

where $\mu_\beta(x, t)$ is company-specific mortality

FSA benchmark: the statistical tests

Test design: Possible μ_β's are specified by parameters $(\beta_1, \beta_2, \beta_3)$

Model for multiple year data:

$$M:\ \mu_\beta(x,t) = e^{\beta_1 r_1(x)+\beta_2 r_2(x)+\beta_3 r_3(x)}\, \mu^{FSA}(x,t)$$

Poisson Likelihood function:

$$L(D,\beta) = \prod_{x,t} \frac{(\mu_\beta(x,t)E(x,t))^{D(x,t)}}{D(x,t)!} e^{-\mu_\beta(x,t)E(x,t)}$$

(This is a GLM-model; implemented in most statistical software.)

FSA benchmark: statistical test hierarchy

Four models:

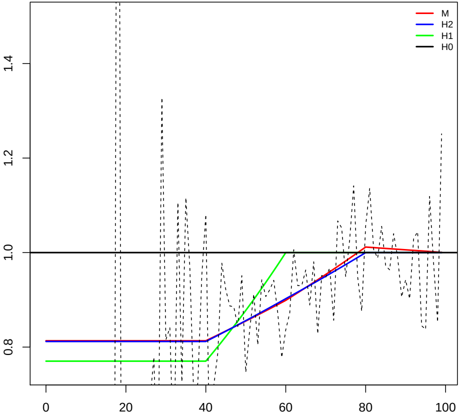
M: $(\beta_1, \beta_2, \beta_3) \in \mathbb{R}^3$
M_2: $\beta_3 = 0$
M_1: $\beta_2 = \beta_3 = 0$
H_0: $\beta_1 = \beta_2 = \beta_3 = 0$

Deviation from benchmark:

$$e^{\beta_1 r_1(x) + \beta_2 r_2(x) + \beta_3 r_3(x)}$$

Series of tests:

1. H_0 vs M
2. M_2 vs M
3. M_1 vs M_2
4. H_0 vs M_1



Longevity stress in Solvency II

Background:

- Difficult to calibrate joint level for longevity risk for all EU countries (99.5% quantile with one-year time horizon)
- Until 2010: 25 percent
- Now: 20 percent stress, in addition to best estimate

News

Danish FSA has "substantial concerns" over longevity calibration for Solvency II

Published online only

Source: Insurance Risk
Author: Aaron Woolner, Laurie Carver
Source: Insurance Risk | 10 Jun 2010
Categories: Solvency II
Topics: Denmark, Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS), Longevity

Discussions in 2010:

...Parner said that with life expectancies increasing – on average – by three years in the last two decades, Ceiops' approach of a uniform 25% stress for countries that updated their tables every 10 years was a good one, "but [it's] a bit too much for those that update on an annual basis"

Longevity stress in Solvency II

Standard model:

- Constant stress (not age-dependent)

Documentation for calibration of longevity stress

- Ceiops' "Solvency II Calibration Paper", 15.4.2010
- Background seems unclear
- Mortality tables not updated yearly in all countries?

> 3.283: "CEIOPS leaves the longevity stress unchanged because historic improvements in mortality rates observed in many countries are sometimes higher than 25% and, according to QIS4 report, the median stress in internal models equals 25%, with an interquartile range of 1% to 25%"

Paper by Lars Hougaard Hansen, 2010:
Assessment of the VaR(99.5%) for longevity risk
Conclusion: Stress of 10–15% is conservative

Challenges with the longevity stress in Solvency II

Same longevity stress for all countries:

- Considerable differences in methods across the EU
- Stress seems to be based on updating every 5 or 8 years
- In DK, the longevity benchmark is being used
- Conclusion: 20% seems to be too big for Denmark

Possible adjustments:

- Full internal model, including longevity risk
- A partial internal model for longevity risk
- **Not possible** with country-specific parameters in the standard model

The Danish FSA suggests we develop a model for the whole sector

Contents

The Danish Society of Actuaries – working group

Task: Develop suggestion for partial internal model for longevity risk ("Danish sector solution"?)

The working group:

- 10 members, one representative of FSA
- First meeting, June 1, 2012.
- Report "Longevity stress and the Danish longevity benchmark", September 18, 2012

Wish list for the model:

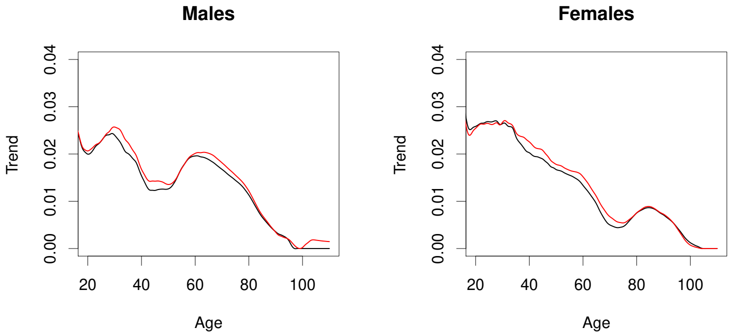
- Reflect risk in Danish longevity benchmark
- Address unsystematic risk (via company specific parameter)
- Simple and easy to understand
- Enter FSA pre-approval process for internal models

Further process:

- Danish Insurance & Pension, FSA, individual companies

Update of trend in FSA benchmark

Trend is updated yearly, e.g. 2011 to 2012:



Trend increased from the “1981-2010” trend (black) to the “1982-2011” (red) trend

Model quantifies risk associated with yearly updating procedure

Update of current mortality in FSA benchmark

Decrease in benchmark for the current mortality from 2011 to 2012

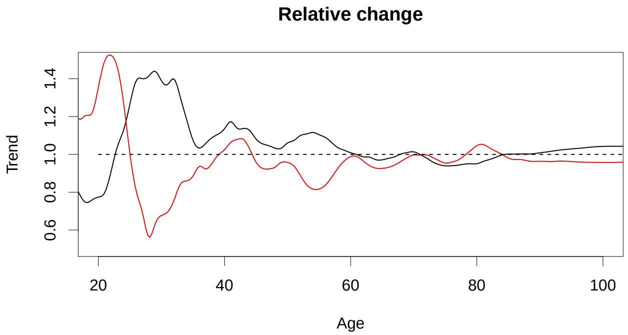


Figure: Relative change, corrected for expected improvements. Females (red) and males (black)

Sensitive to changes in data/population (number of companies included).

Goals and results

Longevity risk in the Solvency II directive, article 105, 2(b):
"the risk of loss, or of adverse change in the value of insurance liabilities, resulting from **changes in the level, trend, or volatility of mortality rates**, where a decrease in the mortality rate leads to an increase in the value of insurance liabilities (longevity risk)"

The Danish FSA divides the longevity risk into two parts:

- **Systematic risk:** Risk associated with changes in the **level** and **trend** for the company's mortality
- **Unsystematic risk:** Uncertainty in the **estimation** of the company's mortality

$$\mu_\beta(t,x) = e^{\beta_1 r_1(x)+\beta_2 r_2(x)+\beta_3 r_3(x)} \cdot \mu^{FSA}(x,t_0) \cdot (1-R(x))^{t-t_0}$$

The proposed model describes these risks! Further:

- adjustment, to mimic model-changes of the FSA-benchmark
- expert-judgment addition, to account for non-quantifiable risks

Contents

Working group – Individual companies via Insurance & Pension

Task: Prepare the suggested model for use as a partial interal model for the sector

- A lot of companies participated in the beginning
- SAS Institute participated, and implemented the model
- Pre-approval process with the Danish FSA

The group has worked well over the years: active participation from a lot of companies

The group still exists, and meets regularly

Collaboration with FSA

What is required of a partial internal model for Solvency II?

- Regular meetings with FSA, to collaborate on the approach
- A lot of governance/documentation - how to approach this?
- In the end: 29 documents, including external literature
  - General description of the model
  - Assumptions of the model
  - Academic paper published in Scandinavian Actuarial Journal (Jarner & Møller, 2015)
  - Code documentation
  - Policy about model changes
  - Policy about validation
  - Use and governance in the company
  - Overview of data
  - Overview of related procedures
  - Overview of documentation
  - ...

Input from FSA during pre-approval process

Requirements from the FSA

- The model is built on the FSA benchmark: necessary to "take ownership" of the benchmark
- FSA requires a company to understand the model in details
- Separate departments for development/calibration and (yearly) independent validation

A lot of companies withdrew. 5 continued.

Changes during the pre-approval process

One of the objectives of the working group of the Danish Society of Actuaries was a model that was "simple and easy to understand"

Change to the "loss-function"

- The original model assessed the 99.5 % quantile in the (marginal) residual lifetimes
- The stress was a best-estimate unisex calibration to this level
- **Question:** Does it accurately reflect different companies individual portfolios?
- **Solution:** A company-specific proxy-portfolio of life annuities

Expert judgment add-on On top of the stress from the model, an add-on was made for non-quantifiable risks (0.5 %)

Contents

Model framework

Consistent modeling of underlying mortalities and data

Framework: (gender-specific, but not included in notation)

National mortality:	$D^N(x,t) \sim \text{Poisson}(\mu^N(x,t)E^N(x,t))$
Sector-specific mortality:	$D^S(x,t) \sim \text{Poisson}(\mu^S(x,t)E^S(x,t))$
Company-specific mortality:	$D^C(x,t) \sim \text{Poisson}(\mu^C(x,t)E^C(x,t))$

Underlying mortalities:

- Systematic risk
- μ^N, μ^S, μ^C are stochastic with given dependence structure

Actual (realized) number of deaths:

- Unsystematic risk
- D^N, D^S, D^C are (conditionally) Poisson distributed.
  Dependent: population overlap assumed.

Development in the mortality: National mortality

Poisson Lee-Carter model (Brouhns et al. (2002))

$$D^N(x,t) \sim \text{Poisson}(\mu^N(x,t)E^N(x,t))$$

where $\log \mu^N(x,t) = a_x + b_x k_t$

- Age-dependent level a_x and "improvement rate" b_x
- Joint improvement index k_t, random walk with drift

$$k_{t+1} = k_t + \varepsilon_{t+1}, \qquad \varepsilon_t \sim N(\xi, \sigma^2)$$

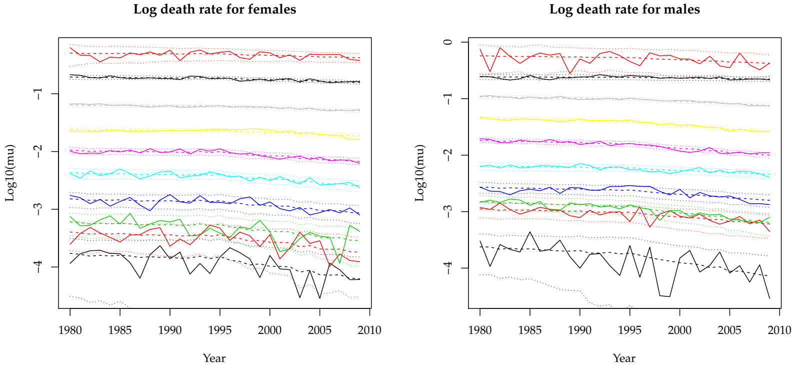
  - Improvements 100% correlated across age
  - Correlation between males and females are estimated
- Realised number of deaths are independent (given μ^N)
- Estimation based on ~~30~~ 20 years of data (as the FSA benchmark)

Simulate one year ahead:

$$\log \mu^N(x, T+1) = a_x + b_x\left(k_t + \varepsilon_{T+1}\right)$$

Poisson Lee-Carter model fitted to Danish data

- Poisson Lee-Carter model fitted to 1980-2009 and age 0-105
- Data (solid line), fit (dashed line), 95% confidence interval (dotted) for age 0, 10, 20, ..., 100
- Unsystematic vol in data in accordance with Poisson variation
- Model reproduces observed drift and volatility in the data



Development in the mortality for sector and company

Assume the development in national, sector- and company-specific mortalities are fully correlated

Sector-specific mortality

- Starting point: most recent mortality benchmark
- Simulate mortality one year ahead (for each gender)

$$\begin{aligned}\mu^S(x, T+1) &= \mu^{FSA}(x, T)\mu^N(x, T+1)/\mu^N(x, T) \\ &= \mu^{FSA}(x, T)\exp(b_x \varepsilon_{T+1})\end{aligned}$$

Company-specific mortality

- Starting point: current company-mortality
- Simulate mortality one year ahead (for each gender)

$$\begin{aligned}\mu^C(x, T+1) &= \mu^C(x, T)\mu^N(x, T+1)/\mu^N(x, T) \\ &= e^{\beta_1 r_1(x)+\beta_2 r_2(x)+\beta_3 r_3(x)}\mu^S(x, T+1)\end{aligned}$$

Simulation of overlapping portfolios

Based on

$$\mu^N(x, T+1), \mu^S(x, T+1) \text{ and } \mu^C(x, T+1)$$

simulate the three independent Poisson random variables

$$D^C(x, T+1), D^{S\setminus C}(x, T+1) \text{ and } D^{N\setminus S}(x, T+1)$$

using suitably adjusted mortalities and exposures.

We assume the company, sector and national populations are contained in each other:

$$\begin{aligned} E^S(x,t) &= E^C(x,t) + E^{S\setminus C}(x,t) \\ D^S(x, T+1) &= D^C(x, T+1) + D^{S\setminus C}(x, T+1) \\ E^N(x,t) &= E^S(x,t) + E^{N\setminus S}(x,t) \\ D^N(x, T+1) &= D^S(x, T+1) + D^{N\setminus S}(x, T+1) \end{aligned}$$

"Simulate the next year"

1. Simulate 100.000 underlying mortalities from the stochastic model for year $T+1$.
2. In each scenario, generate Poisson number of deaths for national, sector and company data
3. Estimate FSA benchmark-trend $\widetilde{R}(x)$: estimated from national data for newest 20 years
4. Estimate FSA benchmark-level $\tilde{\mu}^{FSA}(x, T+1)$: estimated from sector data using a weighted estimation (1,1,1,1,4) from the newest 5 years
5. Estimate company mortality: estimate $\tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3$ using the test hierarchy

In each simulation, we have estimated a new company-mortality for the next year

$$\tilde{\mu}^{model}(x,t) = e^{\tilde{\beta}_1 r_1(x) + \tilde{\beta}_2 r_2(x) + \tilde{\beta}_3 r_3(x)} \tilde{\mu}^{FSA}(x, T+1)(1-\widetilde{R}(x))^{t-(T+1)}$$

Determine the stress

The stress is determined via a proxy for the liabilities

- For each simulation, calculate the liabilities of a proxy-portfolio
  The proxy-portfolio consists, for each gender and age, of a life annuity and a weight. Several proxy-portfolios can be used, e.g. using different interest rates
- Determine the 99.5%-quantile of the liabilities for (each of) the proxy-portfolio
- Numerically find S_{level} and S_{trend} that yields liabilities corresponding to the 99.5% quantile

$$\mu^{model}_{stress}(x,t) = e^{\beta_1 r_1(x)+\beta_2 r_2(x)+\beta_3 r_3(x)} \mu^{FSA}(x,T)(1-S_{level}) \times \left(1 - R(x)(1+S_{trend})\right)^{t-T}$$

(Overparametrized!)

Main results

Stress for systematic and unsystematic risk:

$$\mu^{model}_{stress}(x,t) = e^{\beta_1 r_1(x)+\beta_2 r_2(x)+\beta_3 r_3(x)} \mu^{FSA}(x,T)(1 - S_{level}) \times \left(1 - R(x)(1 + S_{trend})\right)^{t-T}$$

- Approx. calibrated parameters: $S_{level} = 8.0\%$, $S_{trend} = 8.0\%$
- Similar in size to a 10 % mortality reduction
- Quantifies both systematic and unsystematic risk associated with the yearly updating procedure
- The parameters include an expert-judgment addition (0.5 %)

Systematic and unsystematic risk represent 99.5%-quantiles

Total stress - a comparison of life times

Remaining life times in 2019:

Gender	Age	Remaining lifetime	PIM 8 % / 8 %	10 %	20%
Males	20	71.1	1.0	0.7	1.5
	40	48.9	1.0	0.8	1.6
	60	27.2	0.8	0.8	1.6
	80	9.3	0.5	0.5	1.2
Females	20	72.6	1.0	0.7	1.5
	40	50.6	1.0	0.8	1.6
	60	29.1	0.8	0.8	1.7
	80	10.6	0.5	0.6	1.2

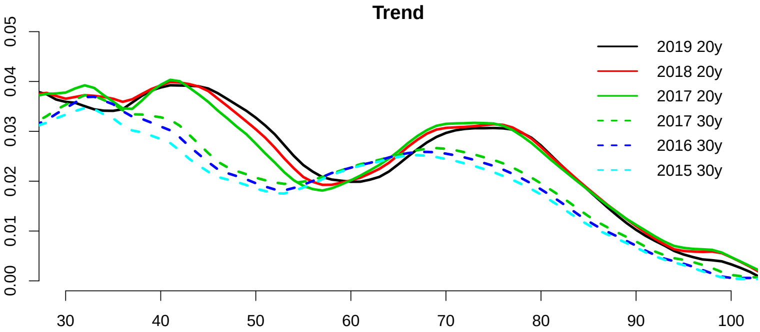
Remaining life times at various ages and increase associated with partial internal model stress, 10 % mortality reduction and 20 % mortality reduction (SII standard model)

Contents

Change of trend data-period from 30 to 20

The FSA benchmark trend: average of 30 years historic improvements

- Updated every year (remove oldest year, add newest year)
- Unstable: trend increased every year from 2011 an onwards



Model risk: The risk of changes to the FSA benchmark trend is not modelled (explicitly) in the partial internal model.

Modelling of the unsystematic risk

If a company experiences an update, which is outside the 99.5 % quantile, they must report swiftly to the FSA.

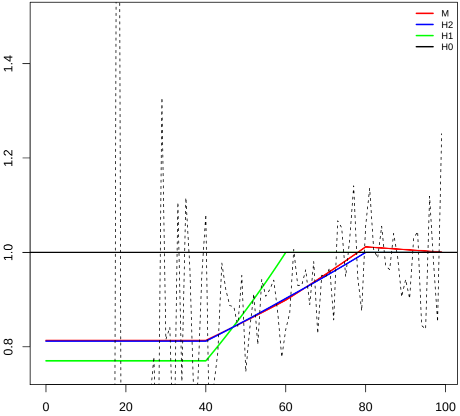
- This happened for a company... and they didn't report it
- The FSA became quite interested...

Modelling of the unsystematic risk

How did it happen?

Recall the deviance: $e^{\beta_1 r_1(x)+\beta_2 r_2(x)+\beta_3 r_3(x)}$

- A "jump" in the test hierarchy occured
- The unsystematic risk was simplified in the original model (test hierarchy wasn't modelled)



Then what?

- The FSA became aware that "not all risks were modelled"
- We had to account for the effect of the lack of modelling the "jump" risk
- The work took 1 year, and the model change was in effect

Main references

Talk based on the original work behind the partial internal model:

- Søren Fiig Jarner & Thomas Møller (2015) A partial internal model for longevity risk, Scandinavian Actuarial Journal, 2015:4, 352-382

Related work:

- Brouhns, Denuit, Vermunt (2002). A Poisson log-bilinear regression approach to the construction of projected lifetables, *Insurance: Mathematics & Economics* **31**, 373–393
- Nielsen (2012). Assessment of longevity risk under Solvency II, *Life & Pension Risk, November issue*, 41–44

Appendix

Development in the mortality: National mortality (cont'd)

(On this slide (only), gender-dependence is included in the notation)

Improvement is a random walk with drift, for each gender g,

$$k_{t+1}^{g} = k_{t}^{g} + \varepsilon_{t+1}^{g}$$

where $(\varepsilon_t^{\mathrm{male}}, \varepsilon_t^{\mathrm{female}}) \sim \mathrm{N}\left(\begin{bmatrix} \xi^{\mathrm{male}} \\ \xi^{\mathrm{female}} \end{bmatrix}, \begin{bmatrix} \sigma^2_{\mathrm{male}} & \rho \\ \rho & \sigma^2_{\mathrm{female}} \end{bmatrix}\right)$

Estimation based on 20 years of data (as the FSA benchmark)

Simulation one year ahead

- Simulate $(\varepsilon_{T+1}^{\mathrm{male}}, \varepsilon_{T+1}^{\mathrm{female}})$, and use, for each gender,

$$\log \mu_g^N(x, T+1) = a_x^g + b_x^g \left(k_t^g + \varepsilon_{T+1}^g\right)$$